

1. $382ab$ is a five-digit number which has remainder 1 when it is divided by 5 or 11. Find the possible values of a .
2. Given that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ and $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
Evaluate $1^3 + 2^3 + 3^3 + \cdots + n^3$
3. If each of three circles, each of radius a , touch the other two. Prove that the area included between them nearly equals $\left(\frac{4}{25}\right)a^2$.
4. Find the value of P if
$$P = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{50^2}\right)$$
5. Find the greatest common factor and the least common multiple of $2\frac{1}{4}$ and $2\frac{2}{5}$.
6. Prove that $(x^2 + y^2 + z^2) > (xy + yz + zx - 2)$ for any real number x, y and z .
7. $M \equiv (8^{39} + 6^{48} + 15^{48} + 20^{39})^{200!}$ is given. Find the smallest non-negative value of x which satisfies $M \equiv x \pmod{7}$.
8. Find the square root of $\frac{3}{2}(x - 1) + \sqrt{2x^2 - 7x - 4}$.
9. Find the equation of the set of points the sum of whose distances from $(4, 0)$ and from $(-4, 0)$ is 10.
10. Prove that $5^{99} + 11^{99} + 17^{99}$ is divisible by 33.