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S/N	EDITION	NAME	HOST COUNTRY	DATE
1	55 <sup>TH</sup>	IMO 2014	SOUTH AFRICA	3 <sup>RD</sup> -13 <sup>TH</sup> JULY
2	45 <sup>TH</sup>	IPHO 2014	KAZAKHSTAN	13 <sup>TH</sup> -20 <sup>TH</sup> JULY
3	46 <sup>TH</sup>	ICHO 2014	HANOI VIETNAM	20 <sup>TH</sup> -29 <sup>TH</sup> JULY
4	25 <sup>TH</sup>	IBO 2014	INDONESIA	6 <sup>TH</sup> -13 <sup>TH</sup> JULY
5	26 <sup>TH</sup>	IOI 2014	TAIWAN	13 <sup>TH</sup> -20 <sup>TH</sup> JULY
6	24 <sup>TH</sup>	PAMO 2014		



**NATIONAL MATHEMATICAL CENTRE**

*"...an international centre for excellence in mathematical sciences"*



**2014  
NIGERIAN  
MATHEMATICS OLYMPIAD  
(JUNIOR)  
2<sup>ND</sup> ROUND  
29<sup>TH</sup> MARCH, 2014**

**Instructions:**

1. Do not open this booklet until told to do so by the invigilator.
2. All working details and explanations must be shown. Answers alone will not be awarded full marks
3. This paper consists of 10 questions for a total of 100 marks.
4. The neatness in your presentation of the solution may be taken into account.
5. Attempt all the questions.
6. You will have 180 minutes working time for this paper.
7. The result will be announced in **[www.nmcabuja.org](http://www.nmcabuja.org)**

1. Find the number of natural numbers less than 100,000 which have 9 as their first digit.
2. Prove that for all  $n \in \mathbb{N}$ ,  $\prod_{k=1}^n (1 + \frac{1}{k}) = n + 1$ .
3. Given that  $x_1, x_2, x_3$  are positive numbers, show that  $(x_1 + x_2 + x_3)(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}) \geq 9$ .
4. The numbers from 1 to 500 are written on a piece of paper. In a single move you choose 2, 3, 4 or 5 numbers from the list, erase them and adjoin to the list the remainder of the sum of the chosen numbers when it is divided by 13. After a number of moves there are only two numbers left on the page. One of them is 102. What is the other?
5. Given that  $\angle A = 45^\circ$ ,  $\angle B = 30^\circ$ ,  
 $\sin(A + B) = \sin A \cos B + \cos A \sin B$ ,  
 $\cos(A + B) = \cos A \cos B - \sin A \sin B$  and  
 $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ .  
 Find  $\tan 15^\circ$ , expressing your answer in surd form.
6. Let  $p$  be a prime number greater than 3. Show that the numerator of the (reduced) fraction  $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{p-1}$  is divisible by  $p^2$ .  
 Hint : consider only two prime numbers.
7. Let  $a$  and  $b$  be distinct real numbers, solve the equation  $\sqrt{x - b^2} - \sqrt{x - a^2} = a - b$ .
8. Let  $P$  be a variable in the interior or on the sides of an equiangular polygon. Prove that the sum of distances from  $P$  to the polygon's side is constant.

9. At what time between 12h00 and 13h00 are the hands of a clock in a straight line?

10. Prove that

$$\frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \dots + \frac{1}{(2n-1)2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n}.$$